Verification of Quantum Light

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Nonclassicality

Multipartite entanglement

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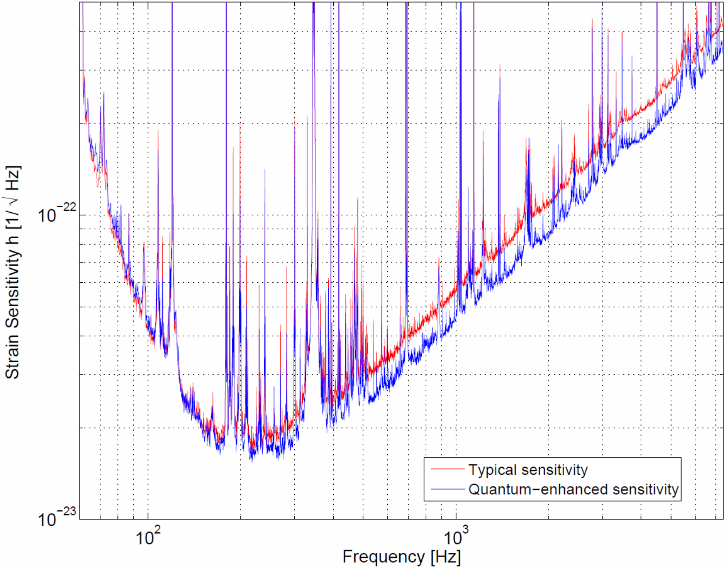
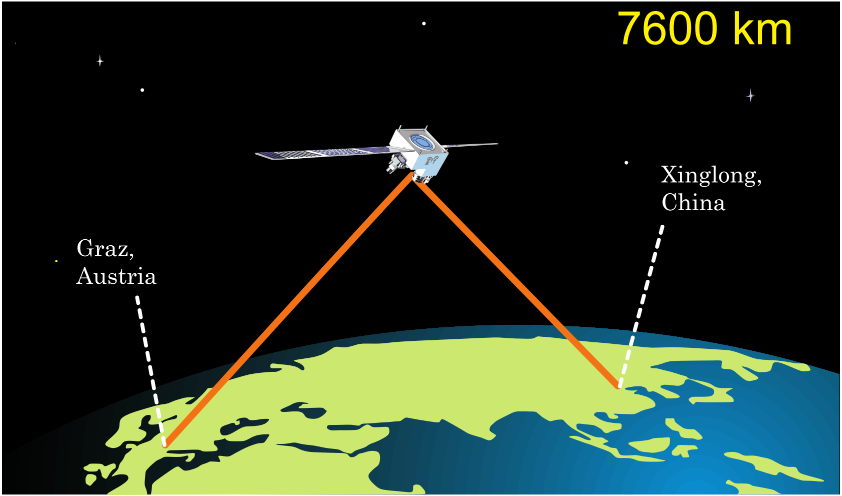
Ligo gravitational wave interferometer

Quantum metrology with squeezed light



Secure communication with quantum light



Present section

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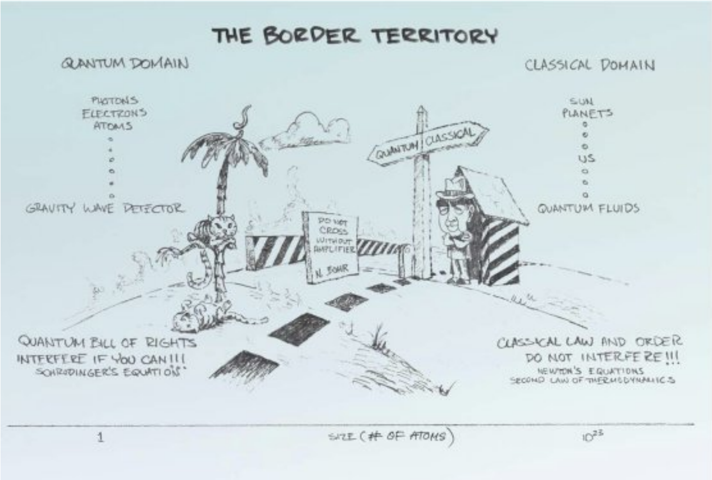
Nonclassicality: quantum superpositions

- Classical reference: coherent states $|\alpha\rangle$
- Nonclassical state: $|\psi\rangle = \sum_i c_i |\alpha_i\rangle$

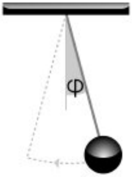


Classical mixtures versus nonclassical states

- Coherent states $|\alpha\rangle$: classical behavior
- Mixture of classical states:

$$\hat{\rho}_{\rm cl} = \sum_i p_i |\alpha_i\rangle\langle\alpha_i| \Rightarrow \int dP_{\rm cl}(\alpha)|\alpha\rangle\langle\alpha|$$

- General quantum state:[1] $\hat{\rho} = \int dP(\alpha)|\alpha\rangle\langle\alpha|$
- $P(\alpha) \cong$ quasiprobability: $P(\alpha) \neq P_{\rm cl}(\alpha)$



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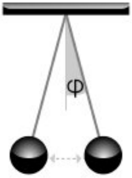

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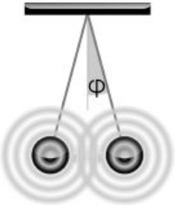

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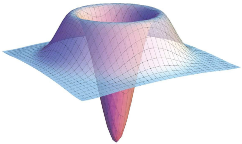

Experimental P function[2]

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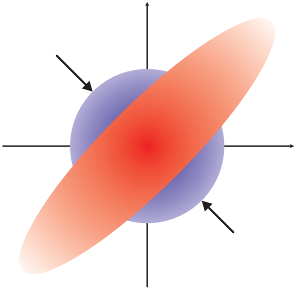
[1] R. J. Glauber, Phys. Rev. **131**, 2766 (1963); E. C. G. Sudarshan, Phys. Rev. Lett. **10**, 227 (1963).
[2] T. Kiesel, W. Vogel, V. Parigi, A. Zavatta, M. Bellini, Phys. Rev. A **78**, 021804(R) (2008).

P function of squeezed state

- Squeezing below the vacuum noise level:



- P function of squeezed vacuum:

$$P_{\mathrm{sv}}(\alpha) = e^{-\frac{V_x - V_p}{8}\left(\frac{\partial^2}{\partial\alpha^2} + \frac{\partial^2}{\partial\alpha^{*2}} - 2\frac{V_x + V_p - 2}{V_x - V_p}\frac{\partial}{\partial\alpha}\frac{\partial}{\partial\alpha^*}\right)}\delta(\alpha)$$

Nonclassicality quasiprobabilities: P_Ω

- Problem: $P(\alpha)$ is singular $\Leftrightarrow \Phi \equiv \mathrm{FT}(P)$ is not integrable
- Filtering characteristic function:[3] $\Phi_\Omega(\beta) = \Phi(\beta)\Omega_w(\beta)$
- Construction of a nonclassicality filter[4]

$\Rightarrow$ Regularized function $P_\Omega = \mathrm{FT}^{-1}(\Phi_\Omega)$, called nonclassicality quasiprobability:

For any quantum state: $P_\Omega < 0 \quad \Leftrightarrow \quad P < 0$

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P_Ω of squeezed vacuum

- Direct sampling of P_Ω:[5]

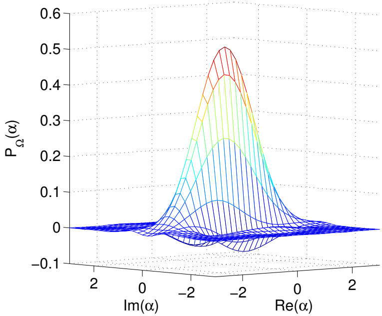
$$P_\Omega(\alpha) \approx \frac{1}{N}\sum_{i=1}^{N} f_\Omega(x_i, \varphi_i; \alpha, w)$$

- Pattern function:

$$f_\Omega(x, \varphi; \alpha, w) = F[\Omega_w(b)]$$

- Phase locked measurement with interpolations
- Continous phase measurement[6]

$\Rightarrow$ Result for P_Ω



Squeezed vacuum state[5]

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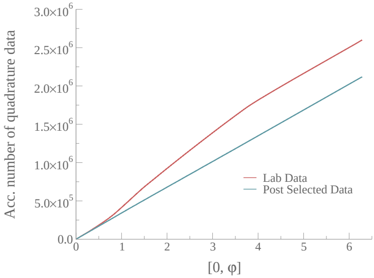

Quantum random numbers

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[6] E. Agudelo, J. Sperling, W. Vogel, S. Köhnke, M. Mraz, and B. Hage, Phys. Rev. A **92**, 033837 (2015).

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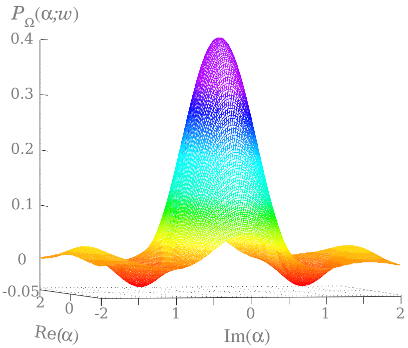

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Example: Schrödinger's cat (1935)

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- Cat state: $|\Psi\rangle \sim |$  $\rangle \otimes |$  $\rangle + |$  $\rangle \otimes |$  $\rangle \neq |\psi_A\rangle \otimes |\psi_B\rangle$

Classical correlation versus quantum entanglement

- Uncorrelated (product) states: $|a, b\rangle \equiv |a\rangle \otimes |b\rangle$
- Mixture of uncorrelated states ⇒ separable states:[7]
  $\hat{\sigma} = \sum_i p_i |a_i, b_i\rangle\langle a_i, b_i|$ (p_i: probability)
  $\Rightarrow \int dP_{\rm cl}(a, b)|a, b\rangle\langle a, b|$ ($P_{\rm cl}$: joint probability)
- General state: $\hat{\rho} = \int dP(a, b)|a, b\rangle\langle a, b|$
- Entanglement quasiprobability:[8]
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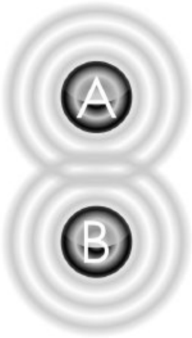

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Nonclassicality versus entanglement

- An N-mode state of light, $\hat{\rho}_{\rm cl}$, is called classical if it can be written as[9]

$$\hat{\rho}_{\rm cl} = \int \mathrm{d}P(\boldsymbol{\alpha})\, |\boldsymbol{\alpha}\rangle\langle\boldsymbol{\alpha}|\,, \text{ with } |\boldsymbol{\alpha}\rangle = |\alpha_1\rangle \otimes \ldots \otimes |\alpha_N\rangle \text{ and } P \equiv P_{\rm cl} \geq 0$$

- An N-partite state $\hat{\sigma}$ is called separable if it can be written as[10]

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- State $\hat{\rho}$ is **nonclassical** / **entangled** if $\hat{\rho} \neq \hat{\rho}_{\rm cl}$ / $\hat{\sigma}$:

$$P(\alpha) \ngeq 0 \;/\; P(\mathbf{a}) \ngeq 0$$

- General relation: entanglement $\Rightarrow$ quantum correlation
- Potential applications in quantum technologies[11]

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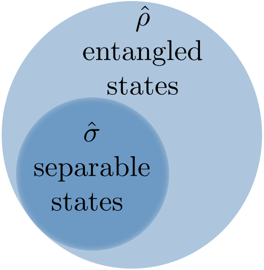
Verifying entanglement – witness operators

- Separable states form a convex set
- Exists hyperplane, $\langle \hat{W} \rangle = 0$, dividing set in two parts; $\hat{W}$: witness operator[12]
- Systematic construction of optimal multipartite entanglement witnesses:[13]
  - Hermitian operator $\hat{L}$
  - Separability eigenvalue problem for N partitions:

$$\hat{L}_{a_1,\ldots,a_{j-1},a_{j+1},\ldots,a_N}|a_j\rangle = g|a_j\rangle, \quad \text{for } j = 1,\ldots,N$$

$$\Rightarrow \quad \hat{W}_{\text{opt}} = \hat{L} - \inf\{g\}\hat{1}$$



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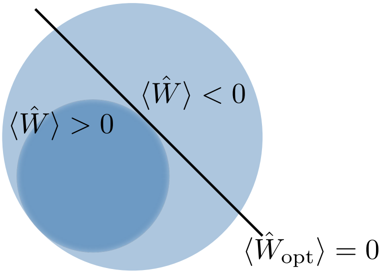

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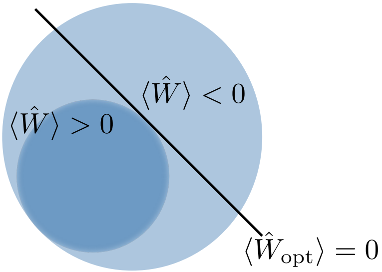

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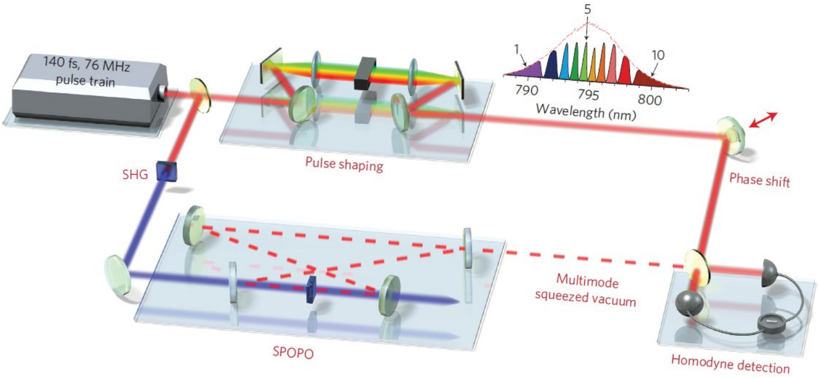
Continuous variable Gaussian entanglement

Synchronously pumped optical parametric oscillator (SPOPO):[14] frequency comb laser. Spectrum divided into elements of equal energy.



[14] J. Roslund, R. Medeiros de Arájo, S. Jiang, C. Fabre, and N. Treps, Nature Photon. **8**, 109 (2014).

Notions of 3-partite entanglement[15]

- Sets of separable states for partitions of same length
- Convex combination of different 2-partitions
- Different notion due to: convex hull≠set union
- Target state:
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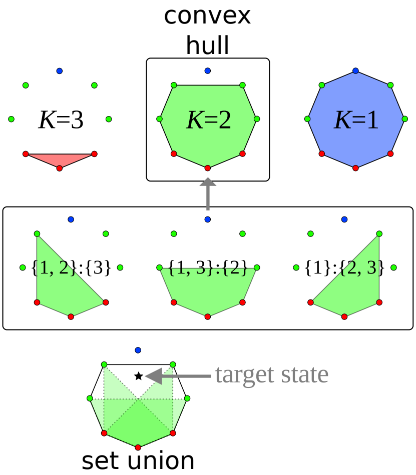
  - not genuine entangled



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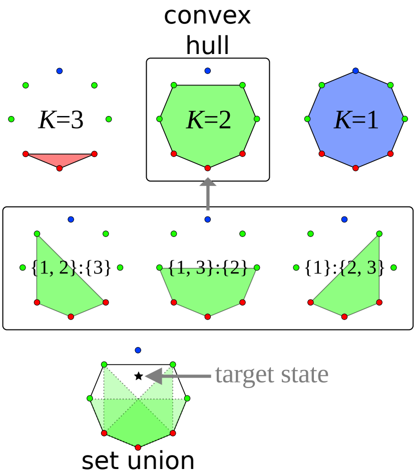

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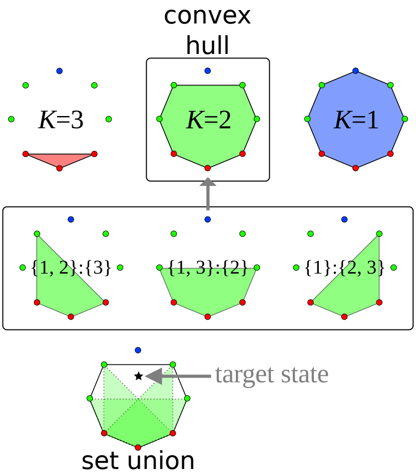

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How genuine is 2-entanglement?[15]

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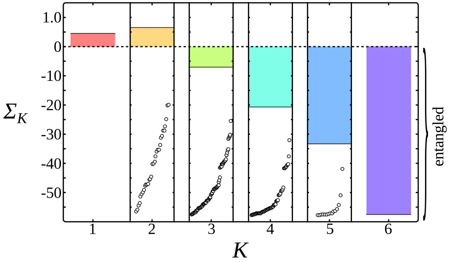
- *K*-Entanglement for $K > 2$
- Absence of "genuine entanglement"

⇒ Entanglement tests for $K > 2$ are indispensable!



- "Genuine test" fails: no insight in highly significant multipartite entanglement
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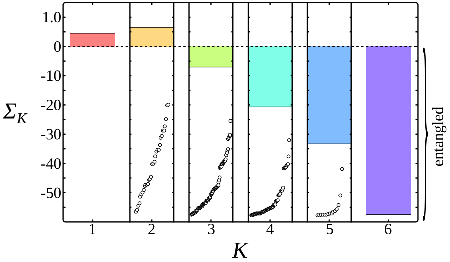

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[15] S. Gerke, J. Sperling, W. Vogel, Y. Cai, J, Roslund, N. Treps, and C. Fabre, Phys. Rev. Lett. **117**, 110502 (2016).

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- K-Entanglement for $K > 2$
- Absence of “genuine entanglement”

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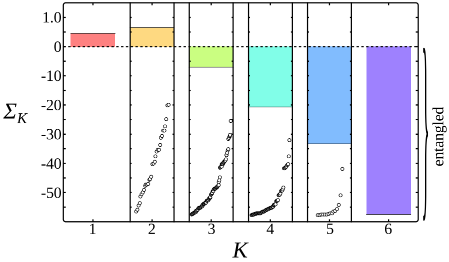

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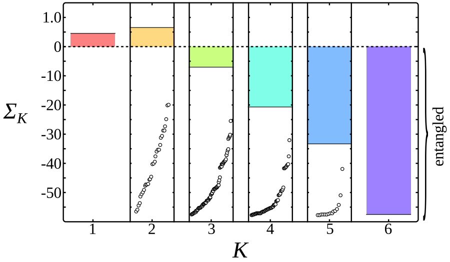

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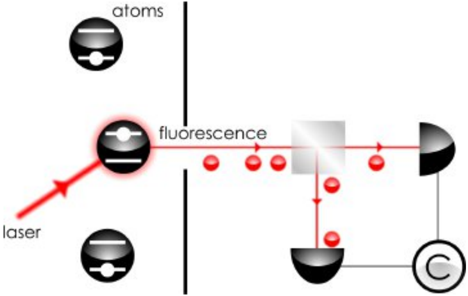
Photon antibunching[16]

- First demonstration of nonclassical light: photon antibunching
- Violation of Schwarz inequality: $\langle \mathcal{T} : \hat{I}(0)\hat{I}(\tau) : \rangle > \langle : [\hat{I}(0)]^2 : \rangle$

$\Rightarrow$ Based on normal- and time-ordered correlation functions!



[16] H.J. Kimble, M. Dagenais, and L. Mandel, Phys. Rev. Lett. **39**, 691 (1977).

General nonclassical field correlations[17]

- Function $P(\boldsymbol{\alpha}) = P(\alpha_1, \ldots, \alpha_N) \Rightarrow P$ functional:

$$P(\{E^{(+)}(i)\}) = \left\langle \mathcal{T} {:} \prod_{i=1}^{k} \hat{\delta}\left(\hat{E}^{(+)}(i) - E^{(+)}(i)\right) {:} \right\rangle, \quad i \equiv (\mathbf{r}_i, t_i)$$

- Nonclassical correlations: $P \ngeq 0$

$\Rightarrow$ Hierarchy of conditions for field correlation functions, such as:

$$|\langle \mathcal{T} {:} \Delta\hat{E}(1)\Delta\hat{I}(2) {:} \rangle| > \sqrt{\langle {:} [\Delta\hat{E}(1)]^2 {:} \rangle \langle {:} [\Delta\hat{I}(2)]^2 {:} \rangle}$$

- Detection: homodyne correlation measurement.[18]
- High-order correlation functions are accessible.[19]

[17] W. Vogel, Phys. Rev. Lett. **100**, 013605 (2008).
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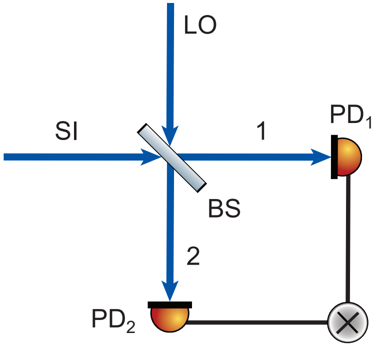
Homodyne cross-correlation measurement[20]

- The measurement setup:



[20] W. Vogel, Phys. Rev. A **51**, 4160 (1995).

Homodyne cross correlation measurement[20]

- Accessible intensity correlation functions:

$$\Delta G^{(2,2)} = \left\{ \left\langle [\hat{E}^{(-)}(\mathbf{r},t)]^2 [\hat{E}^{(+)}(\mathbf{r},t)]^2 \right\rangle - \left\langle \hat{E}^{(-)}(\mathbf{r},t)\hat{E}^{(+)}(\mathbf{r},t) \right\rangle^2 \right\},$$

- Decomposition with respect to local oscillator amplitude, E_{LO}:

$$\Delta G^{(2,2)} = \sum_{i=0}^{4} \Delta G_i^{(2,2)}$$

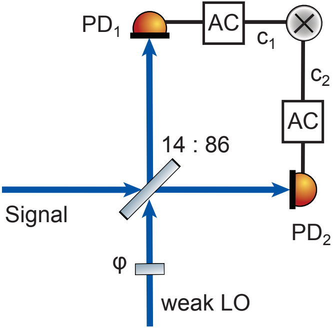
  - Sub-Poisson statistics: $\Delta G_0^{(2,2)} = |T|^2|R|^2 \left\langle : (\Delta \hat{I}_{\mathrm{SI}})^2 : \right\rangle$
  - Anomalous correlation: $\Delta G_1^{(2,2)} = |T||R|(|R|^2 - |T|^2)\, E_{\mathrm{LO}} \left\langle : \Delta \hat{E}_{\mathrm{SI}} \Delta \hat{I}_{\mathrm{SI}} : \right\rangle$
  - Squeezing: $\Delta G_2^{(2,2)} = -|T|^2|R|^2\, E_{\mathrm{LO}}^2 \left\langle : (\Delta \hat{E}_{\mathrm{SI}})^2 : \right\rangle$

[20] W. Vogel, Phys. Rev. A **51**, 4160 (1995).

Anomalous quantum correlations of squeezed light[21]

- Homodyning with unbalanced beam splitter and weak LO[20]
- Classical description
- Correlation $C(\phi) = \langle c_1 c_2 \rangle$ of detector current fluctuations
- Separation of different moments: $C(\phi) = C_0 + C_1(\phi) + C_2(\phi)$
  - $C_0(\phi) \propto \langle (\Delta I)^2 \rangle$
  - $C_1(\phi) \propto E_L \langle \Delta E_\phi \Delta I \rangle$
  - $C_2(\phi) \propto E_L^2 \langle (\Delta E_\phi)^2 \rangle$
- Experimental result $\det[L(\phi)] < 0$:

$$\langle : \Delta \hat{E}_\phi \Delta \hat{I} : \rangle^2 > \langle : (\Delta \hat{E}_\phi)^2 : \rangle \langle : (\Delta \hat{I})^2 : \rangle$$



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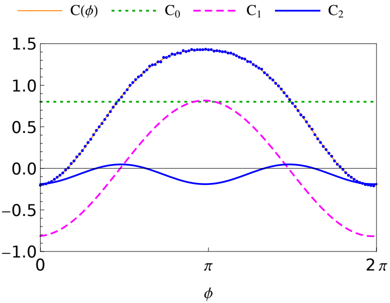

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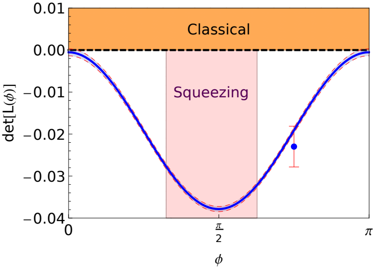

Quantum correlation for large phase intervall!

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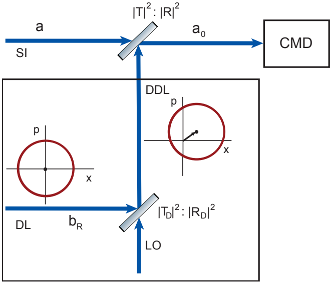
Unbalanced homodyne correlation measurement[22]



- Reference field is a displaced dephased laser (DDL)
- ac correlation of $M = 2m$ detectors yields moments $\langle : [\hat{n}(\alpha)]^m : \rangle$
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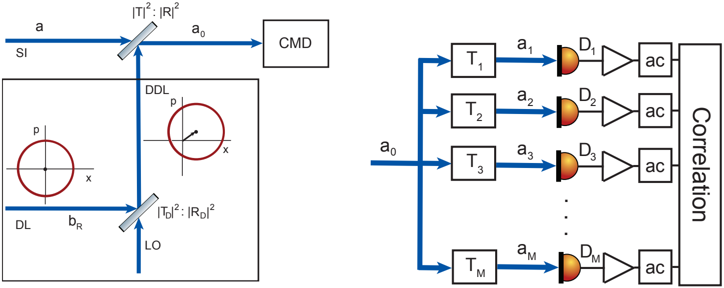

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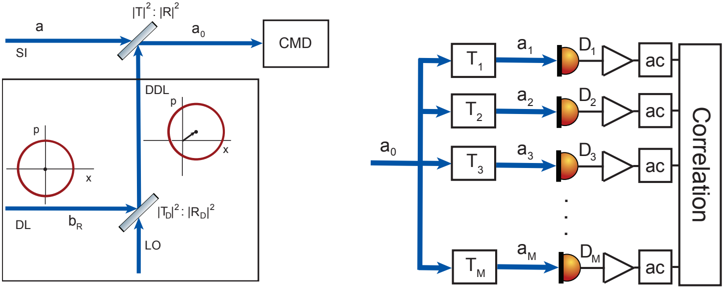

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Click phase-space functions[23]

- Unbalanced homodyning with click detectors
- Introducing click phase-space functions

$$P_N\,(\alpha; s) = \frac{2}{\pi(1-s)} \sum_{k=0}^{N} \left[\frac{\eta(1-s)-2}{\eta(1-s)}\right]^k c_k\,(\alpha; \eta)$$

- N is number of detection bins; for $N \to \infty$

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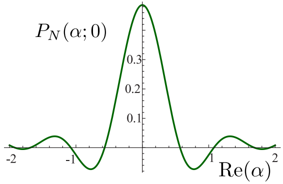
- Negativities in $P_N(\alpha; s)$ ⇒ quantum effects

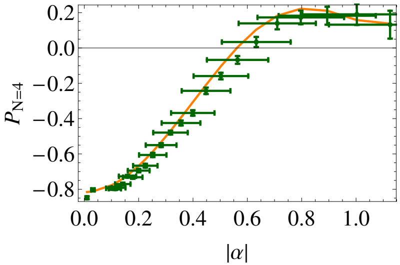


squeezed vacuum state $N = 6$

[23]Luis, Sperling, and Vogel, Phys. Rev. Lett. **114**, 103602 (2015).

Lossy single-photon state from experimental data[24]

Phase-space function $P_N(\alpha; 0)$



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- Quantum efficiency $\eta \approx 0.21$

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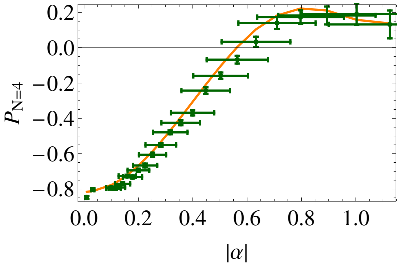

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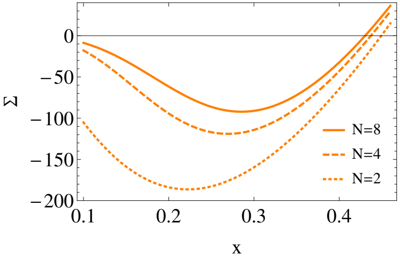

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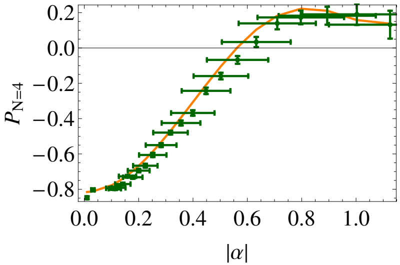

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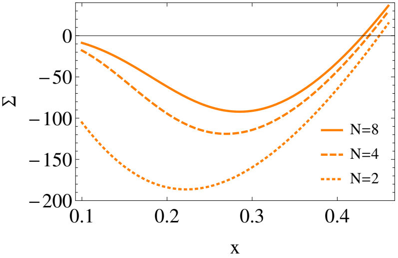

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⇒ Improved verification with fewer detectors

[24] Bohmann, Tiedau, Bartley, Sperling, Silberhorn, and Vogel, Phys. Rev. Lett. 120, 063607 (2018).

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- Nonclassicality quasiprobabilities

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- Entanglement beyond the convex hull of bipartitions
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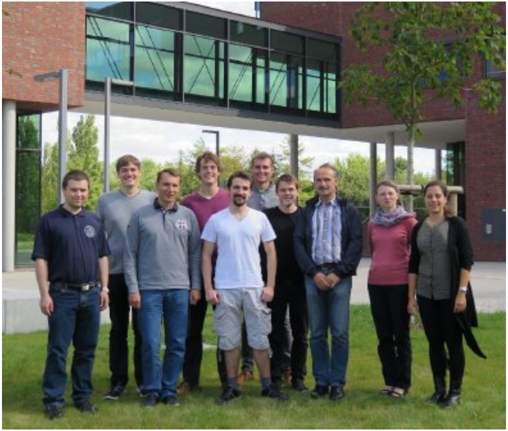
Our new institute

The research group



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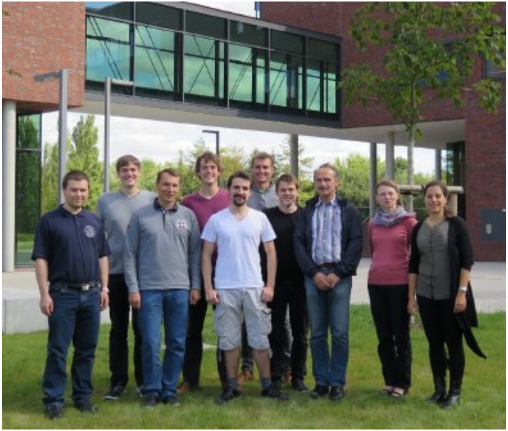

Thank you for your attention!